Limit of a Sequence

Definition: Convergent Sequences

|  |  |
| --- | --- |
| A sequence is convergent to | This means we have a sequence of numbers ,,, that eventually gets very close to a particular number *A* as we move further along the sequence. |
| if for all | No matter how small of a positive number we choose (denoted as ), |
| there exists an | There exists a point (denoted as ) in the sequence |
| such that | For each term of the sequence, the absolute difference between and the limit (denoted as *)* is smaller than the chosen positive number . |
| for all . | This condition holds true for all indices greater than or equal to . |

Definition: Convergent Sequences

*A sequence of real numbers () is said to converge to a real number if:*

, , such that implies

|  |  |
| --- | --- |
|  | “for all greater than zero” |
|  | "there exists some in the set of natural numbers," |
| such that implies | if is greater than , then the absolute difference between and the limit (denoted as ) is less than |

**Definition: Monotone Convergence Theorem (MCT):**

*If a sequence ( ​) is monotone (either non-decreasing or non-increasing) and bounded, then it converges.*

*1. The sequence ​ is decreasing.*

*2. The sequence ​ is bounded below.*

Let *( ​)* be a sequence of real numbers.

increasing

If *( ​)* is monotone non-decreasing

(i.e., *​ ​*for all *​*) and bounded above,

or

if *( ​)* is monotone non-increasing

decreasing

(i.e., *​ ​*for all *​*) and bounded below,

then the sequence *( ​)* converges to a limit.

**Definition: Monotone Convergence Theorem (MCT):**

1. If *( ​)* is non-**decreasing** and bounded **above**,

then there exists a real number such that:

2. If *( ​)* is non-**increasing** and bounded **below**,

then there exists a real number such that:

In both cases, is called the limit of the sequence *( ​)*

**Bounded Below:**

A sequence *( ​)* or a set of real numbers is said to be bounded below

if there exists a real number

such that *​*for all in the sequence

or for all in the set.

*There is a lower bound such that all elements of the sequence or set are greater than or equal to*

**Bounded Above:**

A sequence *( ​)* or a set of real numbers is said to be bounded above

if there exists a real number

such that *​*for all in the sequence

or for all in the set.

*There is a lower bound such that all elements of the sequence or set are less than or equal to*